

Affirming the accepted answer, of what the
Integral of $(\frac{1}{x})$ is supposed to be.

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January 25, 2020

Subject:

According to accepted concepts in Calculus 2, there exists a generalized solution to what the integrals are, of simple power functions of (x) , those being:

$$\int x^p dx, p \neq -1 \equiv \frac{1}{p+1}x^{p+1} + C$$

But, as can plainly be seen, in the case where $(p = -1)$, this leads to the nonsensical conclusion that the integral would be:

$$\frac{1}{0}x^0$$

Yet, there is no reason why the function (x^{-1}) should not have an integral. The accepted answer to this exceptional situation is:

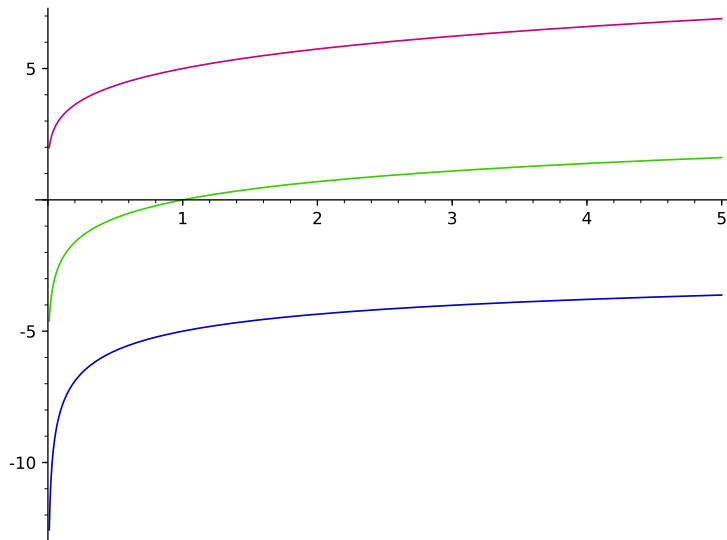
$$\int x^{-1} dx \equiv \ln |x| + C$$

It's assumed that Students who have passed Calculus 2 do not need to be convinced of this. Yet, somebody else could come along and suggest, that this is just a random, silly idea. Therefore, this document will test the premise, by testing a situation which would need to accompany this theory, which is, that "If (p) is close to (-1) , then $(\ln |x|)$ must always lie between the integrals, that will be mere integrals of the corresponding power functions." Even though what results is not absolute proof, what this document will do, is just plot all three functions, when $(0.01 \leq x \leq 5, p \in \{-1.2, -0.8\})$.

$$\begin{aligned}
 F1(x) &= x^{-1.2} \\
 F2(x) &= x^{-0.8} \\
 G1(x) &= (-5) * (x^{-0.2}) \\
 G2(x) &= 5 * (x^{0.2})
 \end{aligned}$$

$$\frac{5}{x^{0.2000000000000000}}$$

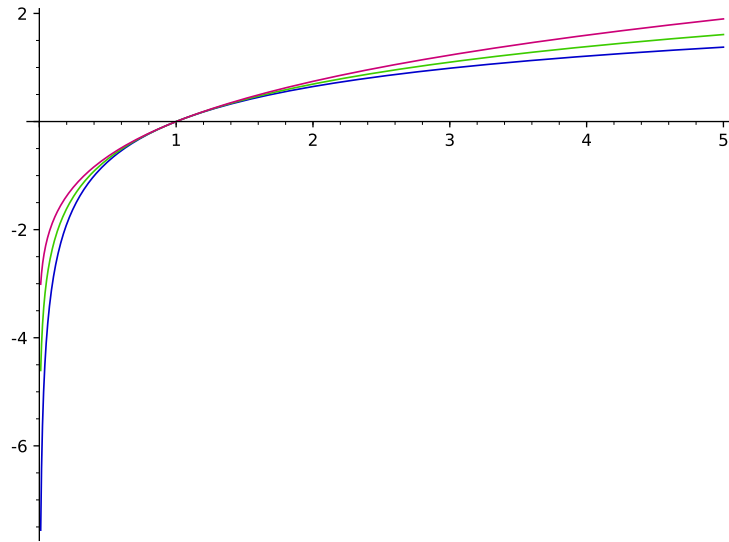
$$5x^{0.2000000000000000}$$



There exists an accompanying premise in the study of integral equations, which states the difference between definite and indefinite integrals, in such a way that any indefinite integral possesses an arbitrary constant, which has been written (C), such that, if the indefinite integral is first computed with it, and then applied at both endpoints of a definite integral, an accurate function of (x) will result as the difference, that is the definite integral. Thus, out of ($G1(x)$) and ($G2(x)$) above, equivalent indefinite integrals can be rewritten, as follows:

$$H1(x) = G1(x) + 5$$
$$H2(x) = G2(x) - 5$$

And then, this will be the resulting plot:



Thus, the premise seems to be consistent, with what the integrals are, when (p) is close to (-1) , but not equal to that value.

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