Attempt to create a perpendicular matrix P for M.

```
—> load(eigen)$
—> M: matrix([+1,0, 2], [0, -2, +1], [2, +1, -1]);
```

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -2 & 1 \\ 2 & 1 & -1 \end{pmatrix} \tag{M}$$

```
—> PL: uniteigenvectors(M);
```

$$[[[-\frac{\sqrt{13}-1}{2}, \frac{\sqrt{13}+1}{2}, -3], [1, 1, 1]], [[[\frac{2}{\sqrt{2\sqrt{13}+13}}, -\frac{\sqrt{13}+3}{2\sqrt{2\sqrt{13}+13}}, -\frac{\sqrt{13}+1}{2\sqrt{2\sqrt{13}+13}}]], [[\frac{2}{\sqrt{13-2\sqrt{13}}}, \frac{\sqrt{1}}{2\sqrt{13} \tag{PL}$$

```
—> PLm: apply('matrix, PL[2]);
```

$$\begin{pmatrix} [\frac{2}{\sqrt{2\sqrt{13}+13}}, -\frac{\sqrt{13}+3}{2\sqrt{2\sqrt{13}+13}}, -\frac{\sqrt{13}+1}{2\sqrt{2\sqrt{13}+13}}] \\ [\frac{2}{\sqrt{13-2\sqrt{13}}}, \frac{\sqrt{13}-3}{2\sqrt{13-2\sqrt{13}}}, \frac{\sqrt{13}-1}{2\sqrt{13-2\sqrt{13}}}] \\ [\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}] \end{pmatrix} \tag{PLm}$$

The problem with PL is, that it doesn't have a list of eigenvectors as its second output element.
It has a list, of lists, of eigenvectors, sorted at top-level per eigenvalue.

There just happens to be one eigenvector per eigenvalue in this case.
This is why, to convert that to a matrix, results in a column vector of lists.

In turn, trying to transpose that, results in a row vector, of lists.

There's nothing wrong in the way the matrix is being built.
It's just useless to me, how the uniteigenvectors() function outputs its eigenvectors.

```
—> P: float(transpose(PLm));
```

$$([0.4448719185337484, -0.7346560672221786, -0.5122201079553044] \quad [0.8312507834516553, 0.12584124303739 \tag{P}$$

```
–>  ReduceDepth(L) :=
    block (
    Output : [],
    for i in L do (
    Output : append(i, Output)
    ),
    return(Output)
    )$
```

```
–>  P: float(transpose(apply('matrix, ReduceDepth(PL[2]))));
```

$$\begin{pmatrix} 0.3333333333333333 & 0.8312507834516553 & 0.4448719185337484 \\ 0.6666666666666666 & 0.1258412430373975 & -0.7346560672221786 \\ -0.6666666666666666 & 0.5414666347632251 & -0.5122201079553044 \end{pmatrix} \quad \text{(P)}$$

The order of columns has been reversed, but the result will be equivalent.

```
–>  D: transpose(P).M.P;
```

$$\begin{pmatrix} -3.0 & 0.0 & -1.110223024625156 \cdot 10^{-16} \\ 0.0 & 2.302775637731994 & -2.220446049250313 \cdot 10^{-16} \\ 0.0 & 0.0 & -1.302775637731994 \end{pmatrix} \quad \text{(D)}$$